COS 702

Assignment 1

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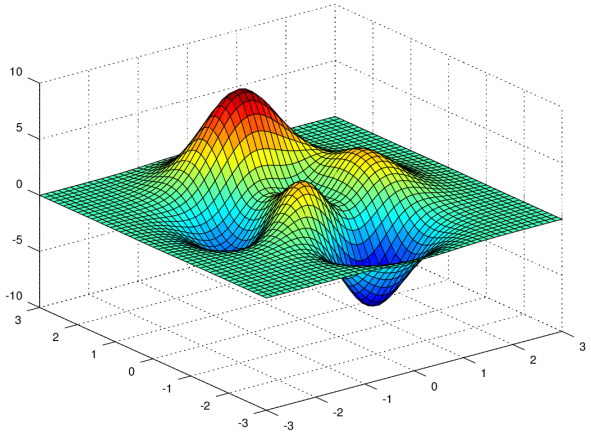
Introduction:

In this assignment, we will be using radial basis functions to approximate scattered data. This process will take a finite set of data points and attempt to reconstruct the surface that contain these data points. The x-axis and y-axis points will be randomly sampled and used to reconstruct the Peaks function.

The Peaks function is plotted as:



The Peaks function is defined as:

$$f(x,y) = 3(1-x)^2 exp(-x^2 - (y+1)^2) - 10(\frac{x}{5} - x^3 - y^5)exp(-x^2 - y^2) - \frac{1}{3}exp(-(x+1)^2 - y^2)$$

Basis Functions:

Various basis functions will be used to reconstruct the data. For the Multi-Quadratic, Inverse Multi-Quadratic, and Gaussian functions, a shaping parameter c is also used. This shaping parameter can be adjusted to get a better fit of the data. The Multi-Quadratic, Inverse Multi-Quadratic, and Gaussian functions will be compared by plotting the Root Mean Squared Error over the shaping parameters. The Root Mean Squared Error of the Polyharmonic Splines function will be compared by plotting the polynomials of the order one up to five.

The basis functions are as follows:

Multi-Quadratic	$\sqrt{(r^2+c^2)}$
Inverse Multi-Quadratic	$\frac{1}{\sqrt{(r^2+c^2)}}$
Gaussian	e^{-cr^2}
Polyharmonic Splines (2D)	$r^2\log(r)$

Data Reconstruction:

To construct the data, 400 x-axis and y-axis points will be generated using the Halton sequence. These x and y coordinates will be passed into the Peaks function to generate the z-axis data. These data points will be used to calculate the weights needed to reconstruct the data. The weights are calculated by converting the data points to a distance matrix and solving an equation in the form of Ac=Z, where A is the distance matrix that holds the distances between each point and all other points, Z is the known value of the Peaks function at that specific x,y point, and c are the weights used to reconstruct the surface.

To reconstruct the data, an evenly spaced mesh grid of size 21 x 21 will be created. These points will be combined with the weights from the previous steps to provide an estimation of the z-axis value of each x and y coordinate in the evenly distributed mesh grid. By multiplying the mesh grid distance matrix A in the form Ac=Z with the previously calculated weights, we get the estimated z value for the x,y mesh grid coordinates.

Data Comparison:

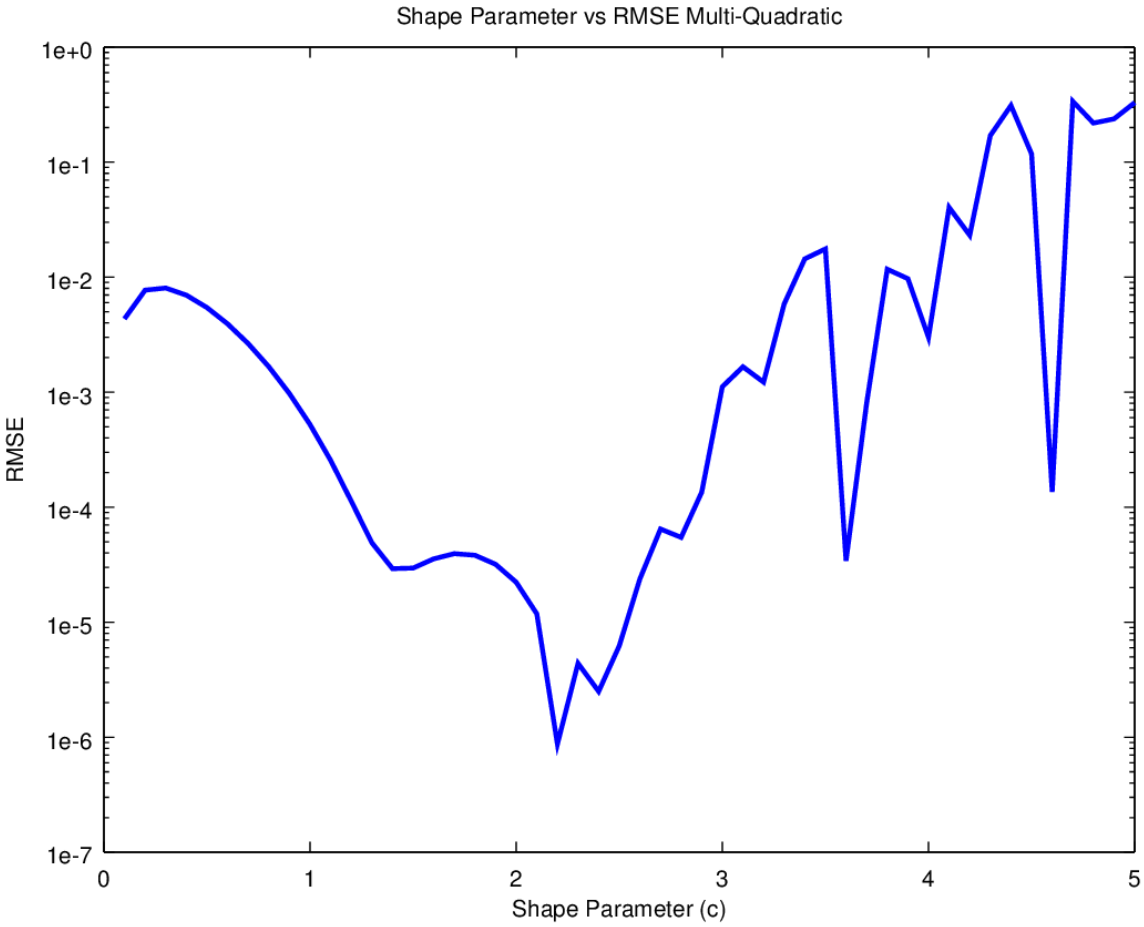
Now that a set of z-axis data has been estimated, we can compare to the true values to see the error. To get the true values for the z-axis, we simply pass the x,y coordinates of the mesh grid to the

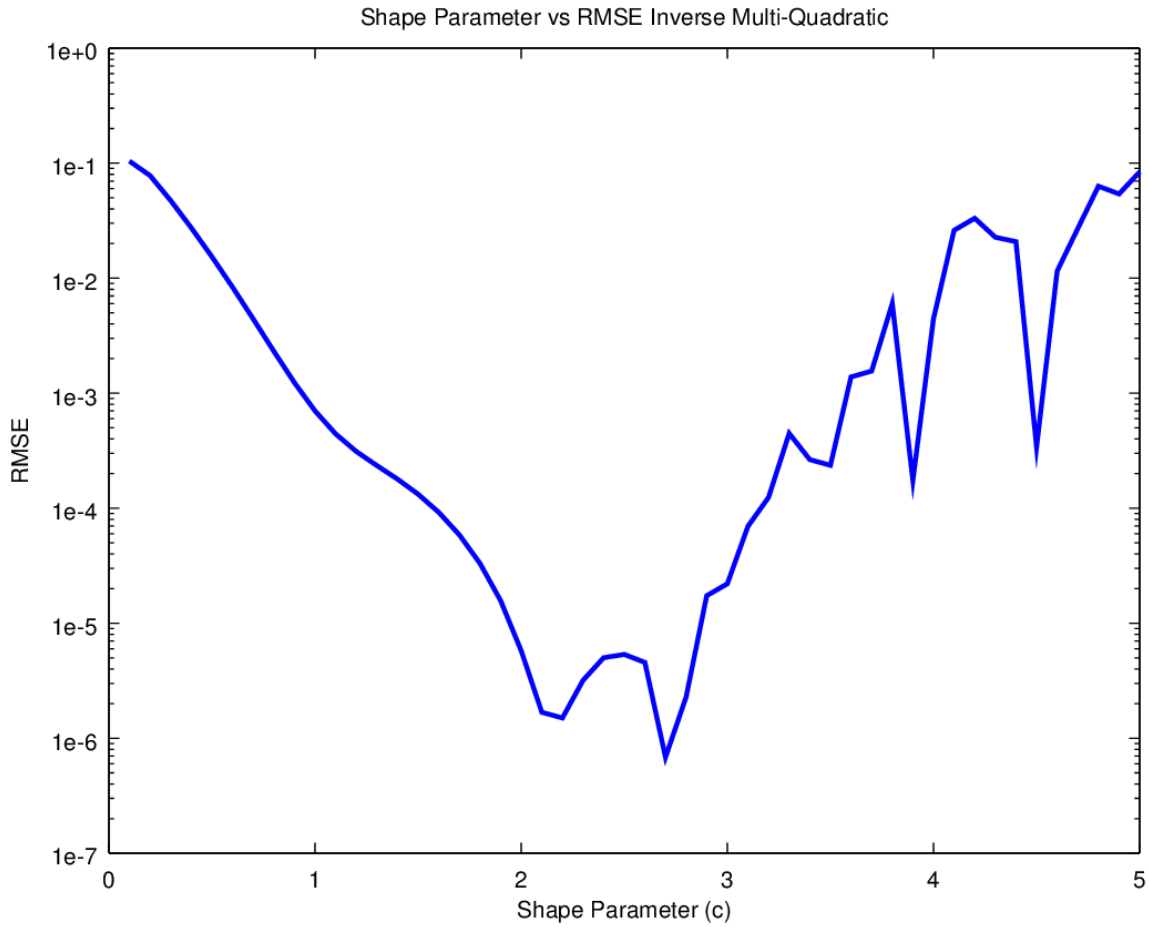
Peaks function, and record the real z values. These real z values can then be compared to the estimated z values. The values will be compared using the Root Mean Squared Error (RMSE). The RMSE is calculated as follows, where Z_r are the real values of Z and Z_e are the estimated values of Z:

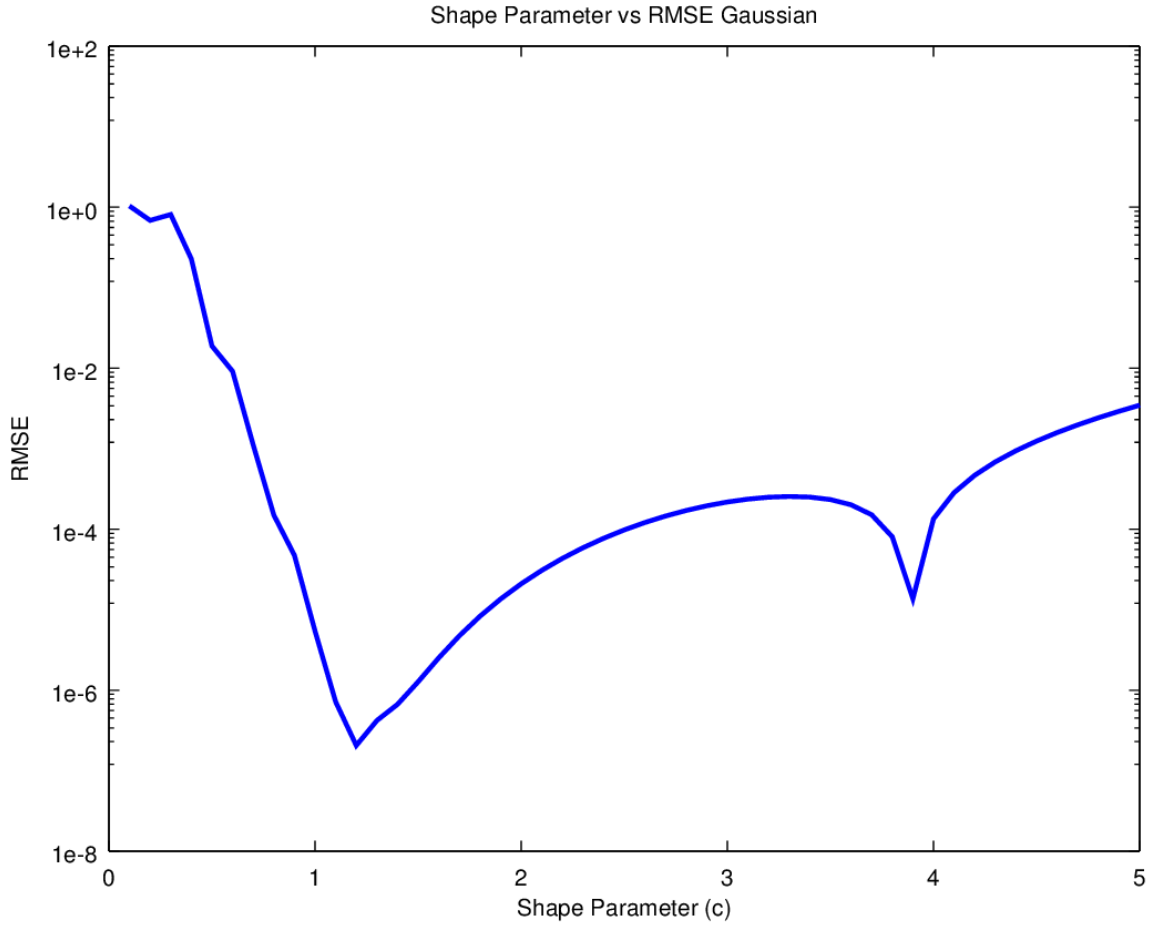
$$\sqrt{\frac{\sum (Z_r - Z_e)^2}{|Z_r|}}$$

Results:

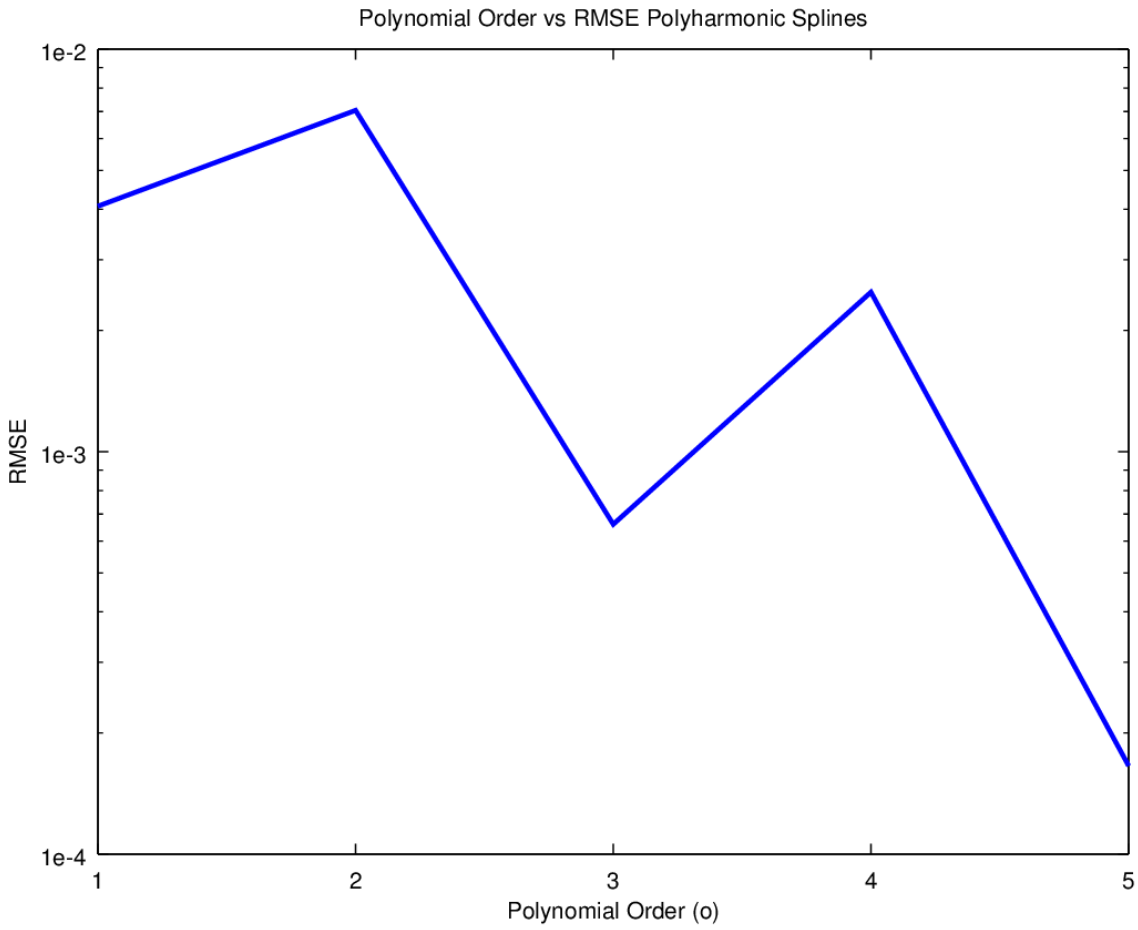
To show the results for the Multi-Quadratic, Inverse Multi-Quadratic, and the Gaussian basis functions, the RMSE values will be plotted vs the shape parameters. For these functions, the shaping parameters from 0.1 to 5 with increments of 0.1 will be used.




Shape Parameter vs RMSE Inverse Multi-Quadratic
RMSE
1e+0
1e-1
1e-2
1e-3
1e-4
1e-5
1e-6
1e-7
0
1
2
3
4
5
Shape Parameter (c)


Shape Parameter vs RMSE Gaussian
RMSE
1e+2
1e+0
1e-2
1e-4
1e-6
1e-8
0
1
2
3
4
5
Shape Parameter (c)

For the Polyharmonic Splines function, there is no shaping parameters. The function is plotted using the polynomial order vs RMSE.



Appendix:

The following contains tables for the actual RMSE data used to plot the above graphs.

Multi-Quadratic Basis	
Shape Parameter	RMSE Value
0.100000	0.004347
0.200000	0.007714
0.300000	0.008050
0.400000	0.006975
0.500000	0.005440
0.600000	0.003926
0.700000	0.002645
0.800000	0.001664
0.900000	0.000973
1.000000	0.000523
1.100000	0.000255
1.200000	0.000113
1.300000	0.000049
1.400000	0.000029
1.500000	0.000030
1.600000	0.000036
1.700000	0.000039
1.800000	0.000038
1.900000	0.000032
2.000000	0.000022
2.100000	0.000012
2.200000	0.000001
2.300000	0.000004
2.400000	0.000003
2.500000	0.000006
2.600000	0.000024
2.700000	0.000065
2.800000	0.000055
2.900000	0.000135

3.000000	0.001116
3.100000	0.001663
3.200000	0.001223
3.300000	0.005851
3.400000	0.014440
3.500000	0.017594
3.600000	0.000109
3.700000	0.000803
3.800000	0.011720
3.900000	0.009705
4.000000	0.000814
4.100000	0.040400
4.200000	0.023155
4.300000	0.170545
4.400000	0.310749
4.500000	0.117938
4.600000	0.000136
4.700000	0.337770
4.800000	0.218837
4.900000	0.237863
5.000000	0.329122

Inverse Multi-Quadratic Basis	
Shape Parameter	RMSE Value
0.100000	0.104087
0.200000	0.077920
0.300000	0.047243
0.400000	0.027356
0.500000	0.015337
0.600000	0.008342
0.700000	0.004422
0.800000	0.002317
0.900000	0.001235

1.000000	0.000701
1.100000	0.000442
1.200000	0.000311
1.300000	0.000235
1.400000	0.000179
1.500000	0.000132
1.600000	0.000092
1.700000	0.000059
1.800000	0.000033
1.900000	0.000016
2.000000	0.000006
2.100000	0.000002
2.200000	0.000002
2.300000	0.000003
2.400000	0.000005
2.500000	0.000005
2.600000	0.000005
2.700000	0.000001
2.800000	0.000002
2.900000	0.000017
3.000000	0.000022
3.100000	0.000070
3.200000	0.000124
3.300000	0.000447
3.400000	0.000264
3.500000	0.000236
3.600000	0.001384
3.700000	0.001558
3.800000	0.006026
3.900000	0.000174
4.000000	0.004442
4.100000	0.026084
4.200000	0.033177
4.300000	0.022688
4.400000	0.020788

4.500000	0.000340
4.600000	0.011534
4.700000	0.027041
4.800000	0.063040
4.900000	0.053983
5.000000	0.083805

Gaussian Basis	
Shape Parameter	RMSE Value
0.100000	1.034148
0.200000	0.685099
0.300000	0.810433
0.400000	0.227520
0.500000	0.018844
0.600000	0.009149
0.700000	0.001115
0.800000	0.000150
0.900000	0.000047
1.000000	0.000005
1.100000	0.000001
1.200000	0.000000
1.300000	0.000000
1.400000	0.000001
1.500000	0.000001
1.600000	0.000003
1.700000	0.000005
1.800000	0.000008
1.900000	0.000014
2.000000	0.000021
2.100000	0.000031
2.200000	0.000043
2.300000	0.000059
2.400000	0.000077
2.500000	0.000098

2.600000	0.000121
2.700000	0.000146
2.800000	0.000171
2.900000	0.000195
3.000000	0.000218
3.100000	0.000236
3.200000	0.000250
3.300000	0.000255
3.400000	0.000250
3.500000	0.000233
3.600000	0.000201
3.700000	0.000151
3.800000	0.000081
3.900000	0.000014
4.000000	0.000134
4.100000	0.000285
4.200000	0.000468
4.300000	0.000686
4.400000	0.000943
4.500000	0.001242
4.600000	0.001586
4.700000	0.001978
4.800000	0.002420
4.900000	0.002916
5.000000	0.003468

Polyharmonic Splines	
Order Parameter	RMSE Value
1	0.004072
2	0.007046
3	0.000660
4	0.002492
5	0.000166