COS 702

Assignment 2

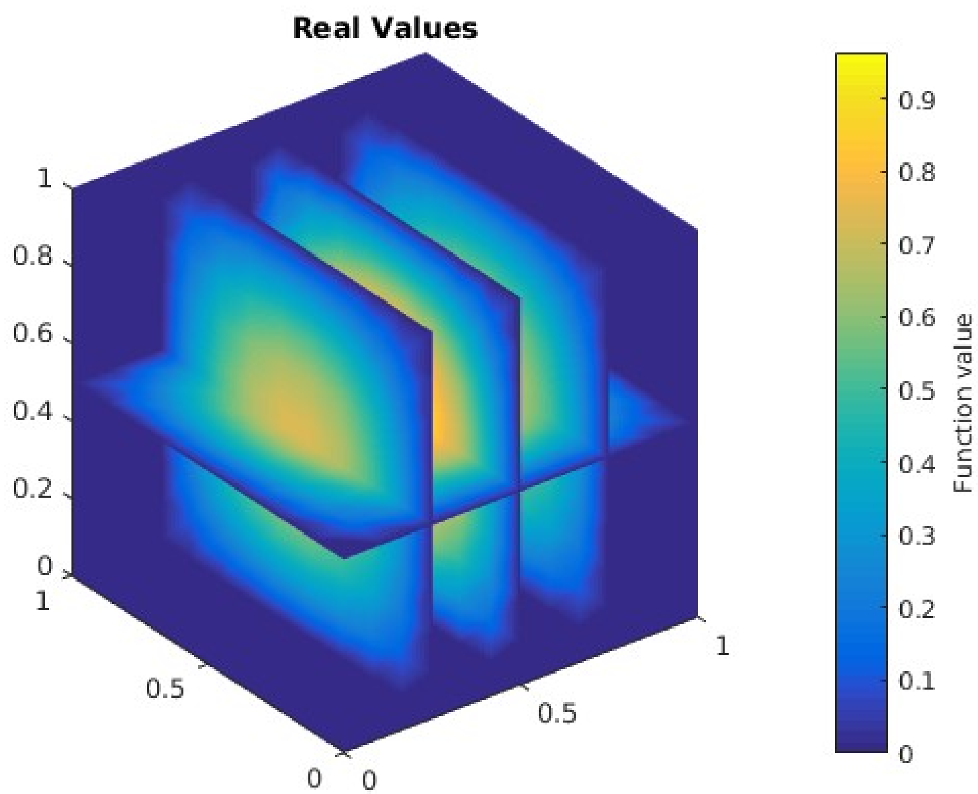
Daniel Lucas Thompson

March 4, 2016

Introduction:

In this assignment, we will be using compact support radial basis functions to approximate data. This process will take a finite set of data points and attempt to provide estimates for other points in this function that are not provided. The function being plotted is a three dimensional function containing coordinates x, y, and z that have a value associated. These x, y, and z coordinates will be used to approximate the following function.

$$f(x,y,z)=64*x*(1-x)*y*(1-y)*z*(1-z)$$



Compact Support Radial Basis Function:

Compact support radial basis functions are a way of speeding up the computation time of data reconstruction. When computing the distance matrix of points, only points within a certain radius are computed. This radius is called the support. Many of the points that are far away are not computed, thus they have a zero value. Considering only these neighboring points that fall within the support radius and using them to create a sparse matrix containing many zeros, we are able to make use of sparse matrix computations.

For this assignment, we will be using one of Wendland's compact support radial basis functions. The function to be used follows.

$$CS-RBF=(1-r)_+^6*(35r^2+18r+3)$$

For a Matlab implementation, it is better to write the function in a shifted form. The shifted form is as follows.

$$CS-RBF(Shifted)=r_+^6*(56*spones(r)-88*r+35*r^2)$$

The purpose of the function spones(r) is to make use of the Matlab sparse matrix optimizations.

Data Reconstruction:

To construct the data, we will be using 729 data points from the file Data3D_729u.mat. These initial 729 3D points will be used to calculate the weights for approximation of the test points. Calculating the weights are done much in the same was as with the global radial basis functions. First, we calculate a distance matrix. However, we only calculate the points that fall within the support radius. This creates a sparse matrix which can be calculated more quickly. After applying the compact support radial basis function to this sparse matrix, we can calculate the weights by solving the equation of the form Ax = b. The value A is the distance matrix with the applied compact support radial basis function. The value of b is the known real value of the x, y, z coordinates that are retrieved using the function we are estimating. The value of x are the weights we are solving for that will be used in the estimation computations.

For data reconstruction, we will be using an evenly spaced mesh grid of size 10 x 10 x 10. These points will be multiplied with the weights from the previous step to provide an estimation of the value of all x, y, z coordinates in the mesh grid. First, we use this mesh grid to calculate a new sparse distance matrix which represents A. Previously, we calculated the value x. Now, we simply solve for b in the form of Ax = b to get the estimated value of all points in the mesh grid.

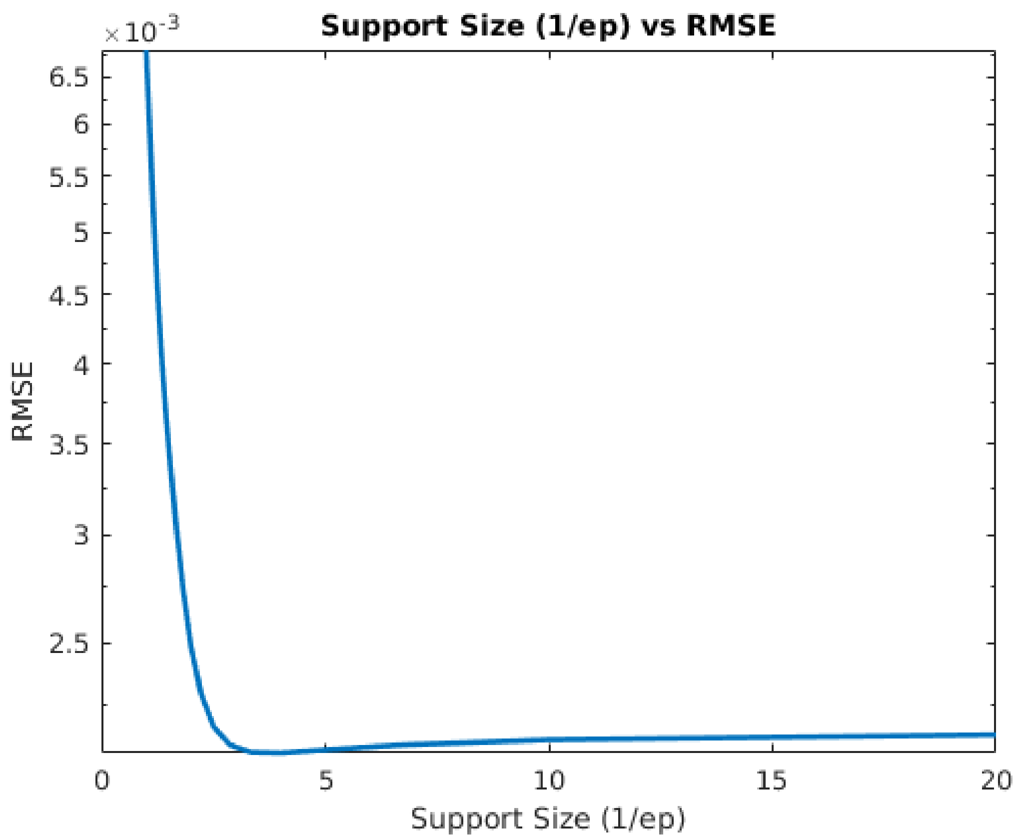
Data Comparison:

For comparing the data, the root mean squared error (RMSE) will be used. To make the comparison, we compute the real values of the 10 x 10 x 10 mesh grid by passing the x, y, and z coordinates to the function that is being approximated. Next, we calculate the RMSE by using these real values and the estimated values computed in the previous section. We will check and plot the RMSE for various support sizes.

The slice graphs of the function for a support size of 1.5 will also be calculated and displayed along with a graph of its absolute error.

Results:

For the RMSE plots, the code takes a parameter *ep*. We determine the size of our supports using the value $\frac{1}{ep}$, and the graph will be plotted with values of *ep* from 0.05 to 1 with increments of 0.05.



Actual values for the Support Size vs RMSE plot graph.

ep	Support Size (1/ep)	RMSE
0.05	20.00	2.140E-03
0.10	10.00	2.122E-03
0.15	6.67	2.104E-03
0.20	5.00	2.087E-03
0.25	4.00	2.076E-03
0.30	3.33	2.078E-03
0.35	2.86	2.105E-03
0.40	2.50	2.171E-03
0.45	2.22	2.294E-03
0.50	2.00	2.483E-03
0.55	1.82	2.739E-03
0.60	1.67	3.044E-03
0.65	1.54	3.372E-03
0.70	1.43	3.711E-03
0.75	1.33	4.089E-03
0.80	1.25	4.520E-03
0.85	1.18	5.020E-03
0.90	1.11	5.589E-03
0.95	1.05	6.195E-03
1.00	1.00	6.804E-03

The estimated plots for support size of 1.5 along with the RMSE plot are as follows.