

COS 702

Assignment 3

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Introduction:

In this assignment, we will be reconstructing a surface using a 3D point cloud. For the point cloud, we will be using a common test model known as the Stanford bunny. This bunny was created by 3D scanning a ceramic rabbit figurine. From this scan, a 3D point cloud of x , y , and z coordinates were created corresponding to points on the surface of the figurine. For the problem in this assignment, we will be using a small 3D point cloud of this bunny that contains 453 points. The original ceramic figurine and point cloud can be seen below.



Surface Construction:

Along with the provided 3D point cloud data, the normal vectors for these data points are also provided. These normal vectors will be used to create extra points to be used in the surface reconstruction. The first step in this process is to remove any normal vectors with a zero value and their

corresponding points. Next, we use the remaining non zero normal vectors to create two extra points for each original data point. We do this by moving a short step in the normal vector direction and creating a new point. We do the same in the negative normal vector direction. We do this for all N original points that didn't have zero normal vectors. This is accomplished with the following equations where N is the number of original points, δ is the step size, and n is the normal vector.

$$(X_{N+i}, Y_{N+i}, Z_{N+i}) = (X_i + \delta n_i^x, Y_i + \delta n_i^y, Z_i + \delta n_i^z)$$

$$(X_{2N+i}, Y_{2N+i}, Z_{2N+i}) = (X_i - \delta n_i^x, Y_i - \delta n_i^y, Z_i - \delta n_i^z)$$

After creating these data points, there are a total of $3N$ data points. We can now use these points along with our radial basis function to compute a set of weights that can be used to reconstruct a surface. First we must set the known value for each of these points. We will do that by setting the original surface to zero, the inner points to -1, and the outer points to 1 by the following.

$$(X_i, Y_i, Z_i) = 0, i = 1, \dots, N$$

$$(X_i, Y_i, Z_i) = 1, i = N + 1, \dots, 2N$$

$$(X_i, Y_i, Z_i) = -1, i = 2N + 1, \dots, 3N$$

Using these lists of points and point values, the weights can be calculated using the form $Ax=b$ where A is the distance matrix with the radial basis function applied for all the points, b are the value of the points computed above, and x are the weights that we are solving for. For this assignment, the global normalized multiquadratic radial basis function is used. It is defined as follows where c is a shaping parameter and r is the distance between the two points.

$$rbf(c, r) = \sqrt{(c*r)^2 + 1}$$

In order to find a good value for c, we will be using fminbnd function which finds the minimum of a variable on a fixed interval. The variables A and b, which represent the distance matrix and values

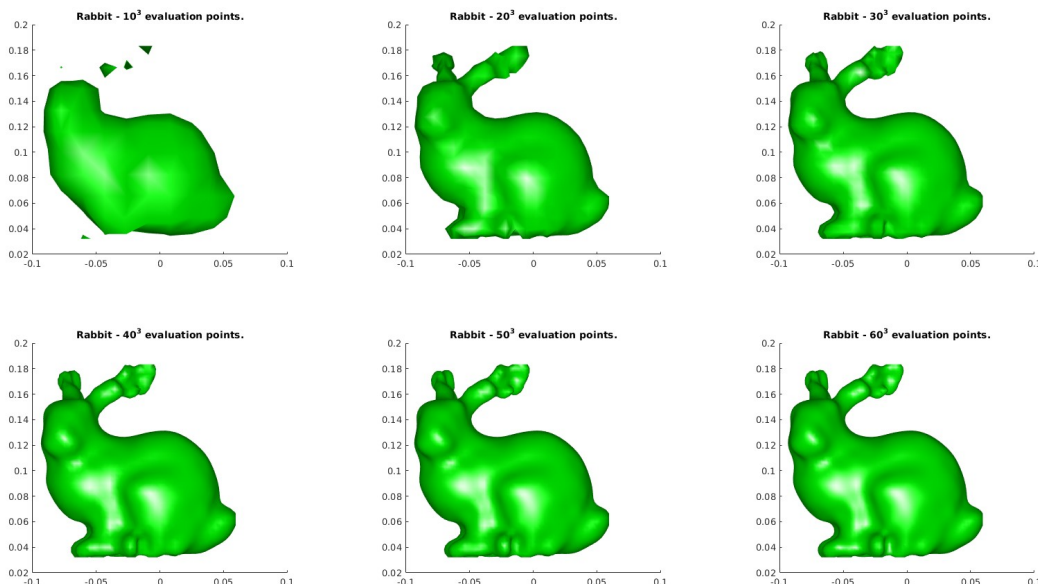
from the previous section, will be passed along with a cost function which is used to calculate the value of each c parameter using the distance matrix and corresponding point values. This `fminbnd` function will then return a good parameter for the shape parameter c which can then be used to reconstruct the surface.

To reconstruct the surface, we must construct a new set of points to be used in making the surface. These points will be created using a uniform mesh grid. Once these points are created, we can combine them with the previously calculated weights to get the values for these points. Once these new points and values have been calculated, we can draw the surface by only drawing the points of the value zero.

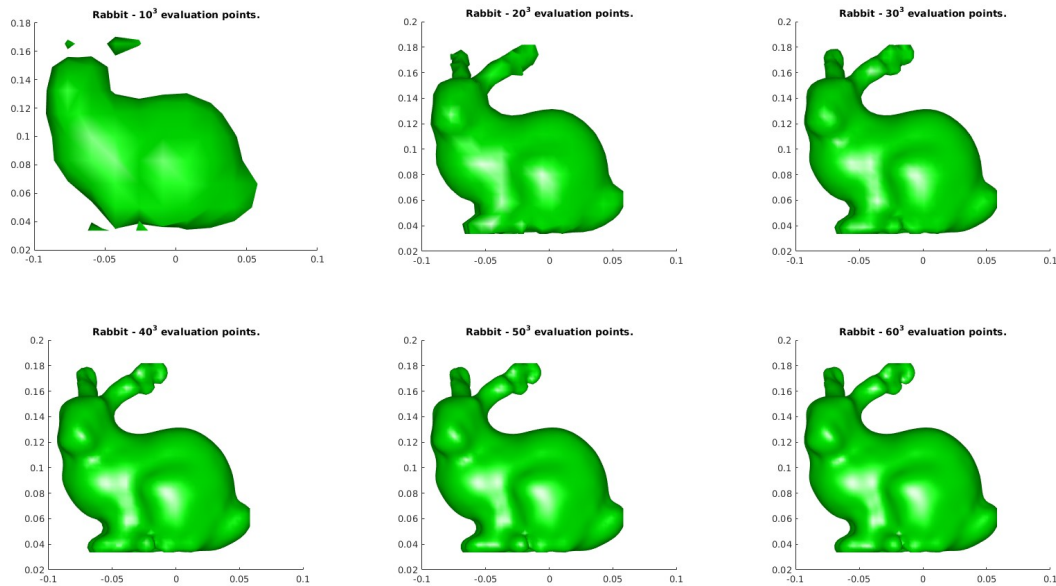
Results:

In trying to reconstruct the surface of this 3D point cloud, the quality of the reconstructed image can be affected by many of the variables. One of the main variables that affect it is the number of mesh grid points used to reconstruct the data. However, the shaping parameter, radial basis function, and normal vector step size also have an effect. For this assignment, the quality of the surface reconstruction will be determined visually.

In the following image, it will be shown how the number of evaluation points can have an effect on the quality of surface reconstruction. For this image, a shape parameter c of 112.230282 was used, with a δ step size calculated by taking the maximum difference between the minimum and maximum original points divided by a constant of 100 which results in a value of 0.001513. As can be seen from the image, at 10^3 the image is very poor and hard to recognize, but as we increase the number of evaluation points, it slowly takes the shape of the Stanford bunny. Once we reach 30^3 evaluation points, further increases provide a visual difference that is negligible.



In the following image, we increase the constant of the δ step size to 10000 which has a value of 0.000015 which results in normal vector points being closer to the original data points. With this change, our new shape parameter c becomes 241.434484. As can be seen from following image, we see the surfaces look similar to the surfaces in the previous images, however, there are slight differences. Overall, both images look very similar to the rabbit.



In the next image, we decrease the constant of the δ step size to 10 which has a value of 0.015131 which results in normal vector points being further away to the original data points. With this change, our new shape parameter c becomes 180.155219. As can be seen from the following image, the quality is much poorer. While the images resemble a rabbit, the rabbits are very lumpy unlike the original rabbit.

